ASSESSING YEAR TO YEAR VARIABILITY OF INERTIAL OSCILLATION IN THE CHUKCHI SEA USING THE WAVELET TRANSFORM

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A

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Abstract

Three years of ocean drifter data from the Chukchi Sea were examined using the wavelet transform to investigate inertial oscillation. There was an increasing trend in number, duration, and hence total proportion of time spent in inertial oscillation events. Additionally, the Chukchi Sea seems to facilitate inertial oscillation that is easier to discern using north-south velocity records rather than east-west velocity records.

The data used in this analysis was transformed using wavelets, which are generally used as a qualitative statistical method. Because of this, in addition to measurement error and random ocean noise, there is an additional source of variability and correlation which makes concrete statistical results challenging to obtain. However, wavelets were an effective tool for isolating the specific period of inertial oscillation and examining how it changed over time.

Introduction

The wavelet transform is a useful and increasingly common analysis tool for investigating non-stationary time series in many different scientific fields. Unlike Fourier analysis, which gives an averaged spectrum over a time series, wavelet analysis provides the ability to examine how the frequency domain of a time series changes over time. Even when compared to short-time Fourier transformation (STFT), the wavelet transform has been shown to give better resolution in both the time and frequency domains (Mallat 1999). Because of its sporadic nature, this advantage makes the wavelet transform a particularly good fit for the examination of inertial oscillation – a rotational motion of water and floating objects in the ocean mainly induced by wind and the Coriolis force (Stewart 2008).

The purpose of this paper is:

- To inform readers of the use of the wavelet transform in the analysis of non-stationary time series, and
- To use the wavelet transform in the analysis of ocean drifter data from the Chukchi Sea.

The Chukchi Sea

The Arctic is especially sensitive to climate change, but relatively little is known about its flora, fauna, and physical processes (Sturm et al. 2003, Grebmeier et al. 2006a,b, Hopcroft et al. 2008). Getting a better understanding of the processes at work in the Arctic and how they are changing is a crucial first step to combating or, at least, adapting to the consequences of climate change.

The Chukchi Sea is a productive and ecologically diverse ocean habitat wedged between northeast Siberia and northwest Alaska (figure 1). Understanding the sea's physical proper-

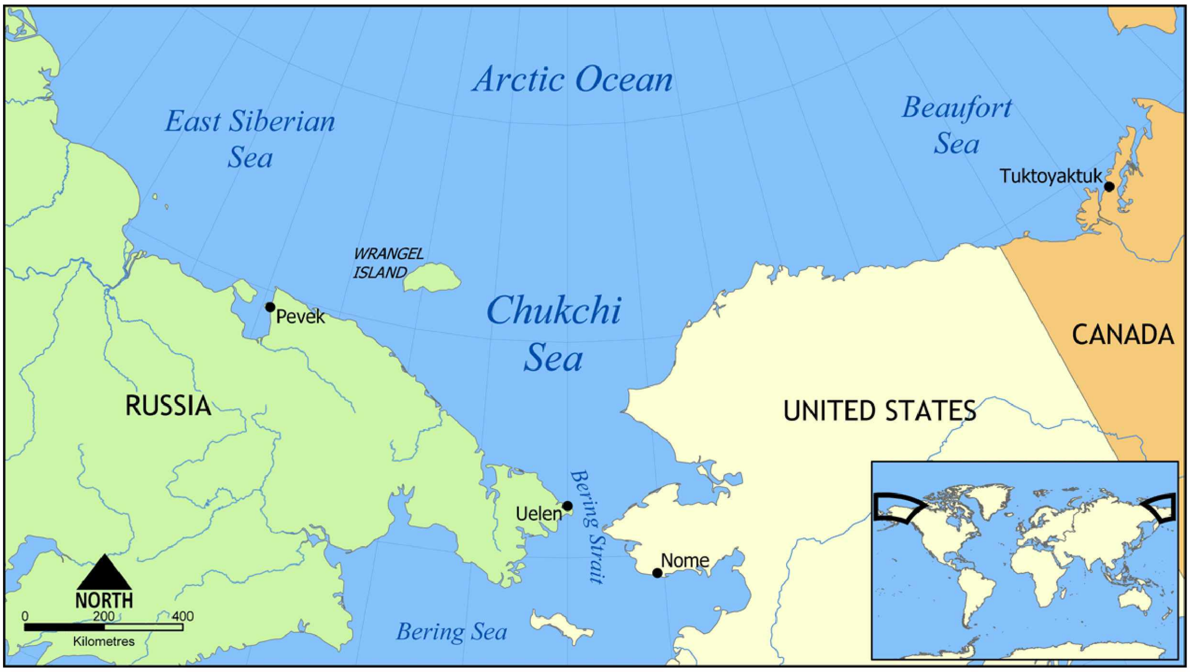

Figure 1: The Chukchi Sea and surrounding area. *Wikipedia.org*

ties can help us better understand how climate change and resource exploitation may affect the area.

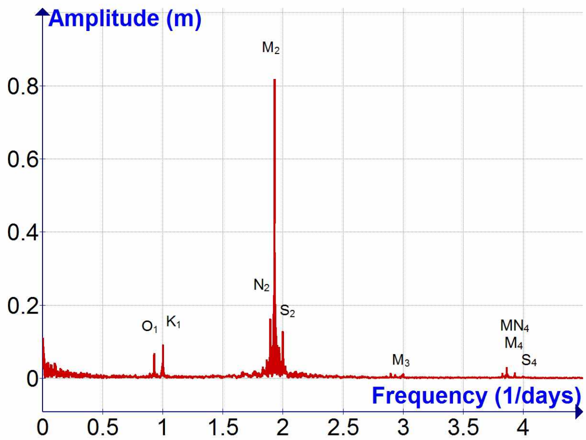

Figure 2: The main tidal constituents and their relative strengths. The frequency of the tidal constituents is given per day. For example, the Lunar semi-diurnal (M_2) tide is the strongest with a frequency of slightly less than twice per day. *Wikipedia.org*

Inertial Oscillation

Tides are the net result of many different influences called tidal constituents (Figure 2), each with its own period. Ocean habitats have very diverse physical characteristics, though all have some form of tidal constituents, which are largely dictated by the gravitational effects of the sun and the moon. However, an area's geography can have a significant effect on which tidal constituents are present and their strengths. The major tidal constituents are the Principal Lunar Semi-diurnal (M_2, period of 12.42 hours) and the Principal Solar Semi-diurnal (S_2, period of 12 hours). The M_2 tide is caused by the gravitational pull of the moon and is the dominant component of the rising and falling tides seen at the coast. The M_2 and S_2 constituents are largely what make up the typical twice daily high and low tides common to most coastal areas. Most of the dozens of tidal constituents go unnoticed as they are barely perceptible even in extremely large data sets.

Inertial oscillation is a movement of water and objects floating in the water that arises from the interaction of the Coriolis force, currents, and wind. It is unlike tidal constituents in that it does not greatly affect the rising and falling tides visible at the ocean's edge. It is a sporadic movement that can have a very strong signal when compared to even the strongest of the constant tidal constituents. Unlike the regular tidal constituents, however, inertial motion is a rotational motion. Inertial oscillation often occurs with such a clear signal that identifying these events is not difficult. However, the far Northern and far Southern oceans and seas present an additional challenge.

Inertial oscillation, like tidal constituents, occurs at a specific frequency. However, the period of inertial oscillation changes with latitude. The period is calculated as:

$$T_I = \frac{2\pi}{f}$$

where

$$f = 2\Omega \sin(\phi)$$

$$\Omega = 7.292 * 10^{-5}\text{seconds}$$

$$\phi = \text{Latitude}$$

We can simplify this to

$$T_I = \frac{12\text{hours}}{\sin(\phi)}$$

This creates a problem in the analysis of high latitude tides in general and obscures sources of variation. The inertial period varies from 12 hours at the poles to *infinity* at the equator. In the region of the Chukchi Sea, 66° to 74° north latitude, the inertial period is very close to the period of the M_2 tide (12.42 hours). Fortunately, the Chukchi Sea has particularly weak M_2 tides, and therefore inertial oscillation easily drowns out the signal of the regular tides. Because of the weakness of the M_2 tide in the Chukchi Sea, it does not show up as statistically significant in Fourier analysis or localized spectral analysis like wavelets. Additionally, since the M_2 tidal constituent is stationary, non-stationary attributes at that frequency would suggest another cause for the strength of the signal.

In the Chukchi Sea, inertial oscillation often manifests as a series of "u" shaped curves that connect at points or small loops such as those seen in Figure 3.

The remainder of this paper is organized as follows. In the next section of this paper, the data will be discussed in more depth, including its source and problems associated with it. Following that the wavelet transform will be explicated using a simplified example then the methods of the analysis will be outlined . In the results section, the major results of the project will be stated and explained. The discussion section is after that in which the results

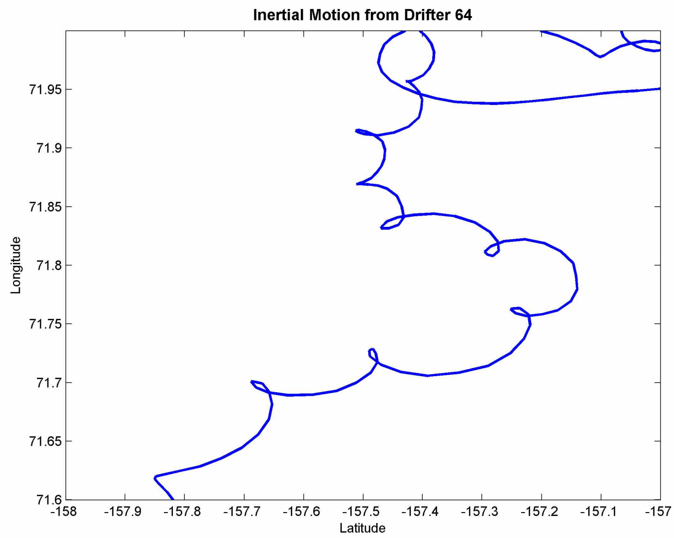

Figure 3: A blown up section of a drifter track showing strong inertial oscillation.

are interpreted. This is followed by the conclusion, which contains some finals thoughts on the project and future work.

Data and Methods

Data

A drifter (Figure 4) is a device that comes in many different forms and is used to gather oceanographic data. The drifters used to gather these data have four arms so that they are pushed around in the water, with floats on the top part of the end of each arm. This keeps the drifters almost entirely underwater so there is no movement that comes directly from the wind. Each drifter has a probe at the bottom of its 1 meter long core which collects data on sea surface temperature, latitude, longitude, east-west velocity, and north-south velocity. The probe collects this data hourly and transmits it via a small antenna at the top of the drifter.

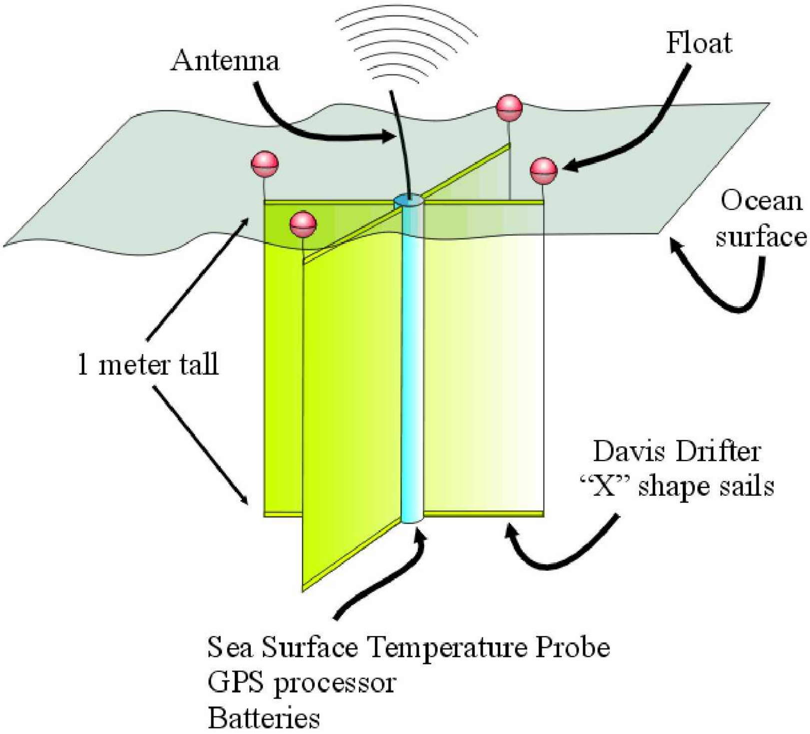

Figure 4: Diagram of a Drifter

During each of the summers of 2012, 2013, and 2014, drifters were released in the Chukchi Sea in several groups of various sizes, usually numbering 20-30. Once released, the drifters go wherever the water takes them collecting measurements along the way. The data is graded on a scale from zero to three based on the number of satellites that are fixed on the given drifter. For example, a drifter that has three satellite fixes gets a quality score of three, which is the highest quality a drifter can have. Additional satellite fixes do not significantly improve accuracy. A drifter that has one satellite fix gets a quality score of one, which is lower quality (higher chance of inaccuracy) than a two or three. If there is no data the score is zero. The data from 2012 contained only data with quality scores of three and had very few gaps. In 2013 and 2014, however, the design of the drifter was modified and much more lower quality data was recorded, so the time series from those years uses data with all non-zero quality scores. Furthermore, for 2013 and 2014, all gaps of less than three hours were automatically linearly interpolated. This approach, however, does not solve the problem of much larger periods of missing data.

Unfortunately, it was not an option to leave missing values as NaN's (Not a Number) in most of the analysis. The wavelet transform does not accept NaN's, so the data used for wavelet analysis (east-west velocity, north-south velocity) could not contain any. However, the primary focus of this research is the periodic behavior of the drifters, and false negatives are preferable to false positives in terms of this periodic behavior. With this in mind, the missing velocity data were designated zero rather than NaN. This removes any periodic behavior in the data, whether at the inertial frequency or otherwise, but it is simple and it allows a wavelet transform to be computed. Furthermore, this convention says there is no inertial oscillation where there is no data to corroborate it.

The Wavelet Transform

The wavelet transform being used for this analysis was done in MATLAB and allows for the estimation of the relative strength of cyclical movement in a given time series at many different periods, i.e. the time it takes for one cycle. The wavelet's scale is defined to give the wavelet function periodic behavior that is equivalent to a specific Fourier period. A convolution is then performed on the scaled wavelet function and the data, which has been centered, producing a vector whose length is equal to that of the data. In the places where the wavelet function and data have similar periodic behavior, the convolution gives larger values. In places where the data do not align well with the wavelet function, the convolution gives smaller values. These "values" are the result of the wavelet transform, and their square is what is commonly referred to as "power" or "wavelet power". The following paragraphs establish notation and provide details, including how to calculate the various plots of power spectra.

Formally, the definition of the wavelet transform is:

$$U_t(s) = \sum_{\tau=0}^{N-1} y_\tau \Psi^* \left(\frac{(\tau - t)\delta t}{s} \right) \tag{1}$$

where

$$y_\tau = \text{a given time series}$$

$$\Psi^* = \text{complex conjugate of } \Psi, \text{ the wavelet function}$$

$$t = \text{a localized time index, or the offset of data and wavelet function } (t = 0, 1, 2, ..., N-1)$$

$$s = \text{wavelet scale, which are determined by analyst before transform}$$

$$\delta t = \text{sampling interval}$$

$$N = \text{the length of the time series}$$

Thus, $U_t(s)$ is a function of the scale s, for each time t.

$U_t(s)$ is the convolution of the discrete times series with a scaled wavelet function. A convolution of discrete series is a mathematical operation in which two series of equal length are paired point by point, each pair is multiplied, and then all multiplied pairs are added together. For example, consider a series of data y_w and a wavelet function Ψ_1:

$$y_w = 0, 1, 2, -3$$

$$\Psi_1 = 1, -1, 0, 0$$

and $U = y_w * \Psi_1$ is the convolution of y_w and Ψ_1.

Then

$$U(1) = (0, 1, 2, -3) \cdot (1, -1, 0, 0) = -1$$

$$U(2) = (0, 1, 2, -3) \cdot (0, 1, -1, 0) = -1$$

$$U(3) = (0, 1, 2, -3) \cdot (0, 0, 1, -1) = 5$$

$$U(4) = (0, 1, 2, -3) \cdot (-1, 0, 0, 1) = -3$$

and

$$U(\cdot) = (-1, -1, 5, -3)$$

Wavelet power is defined as $\mid U_t(s) \mid^2$, noting that $U_t(s)$ may be real-valued or complex (as the wavelet function Ψ may be real-valued or complex).

In the example, the wavelet power, which represents one row on the wavelet transform plot would be:

$$\mid U(\cdot) \mid^2 = (1, 1, 25, 9)$$

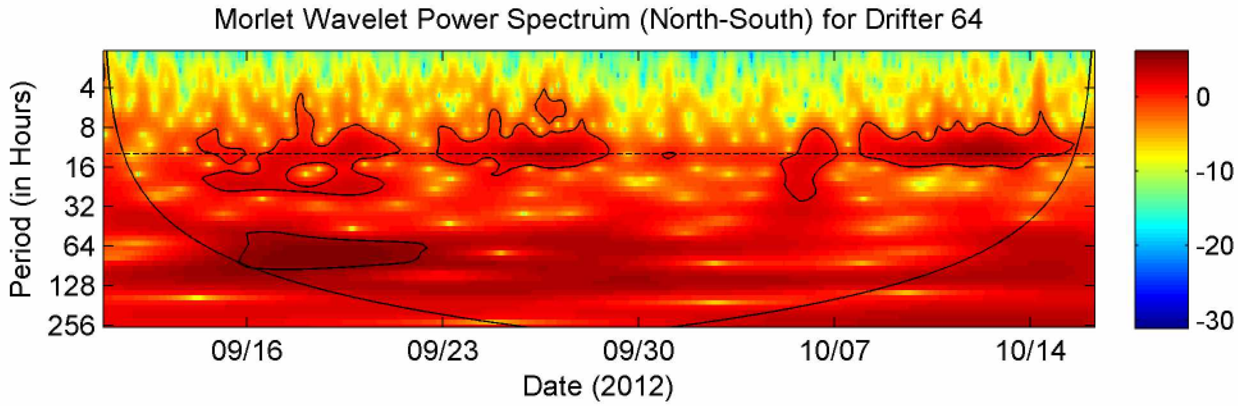

Figure 5: A Morlet wavelet transform using the north-south velocity data from drifter 64 in 2012. The traditional y axis represents period, which is the scale of the wavelet times a constant, while the usual x axis represents time t. The color axis is log_2 of the wavelet power ($log_2(\mid U_t(s) \mid^2)$. This design allows for changing periodic behavior to be estimated over many periods.

A visual representation of this transform using north-south velocity data can be seen in Figure 5. This wavelet transform (Figure 5) has an hourly estimated wavelet power (log_2 of the wavelet power by convention, given as the color axis) for 57 different periods over the course of slightly over one month.

In general when the wavelet function and the data have similar shape (and so period), the negative values in the wavelet function will be multiplied by negative values in the data and positive values will be multiplied by positive values, giving a larger value for power.

Rather than evaluating equation (1) directly the convolution theorem can be applied for computational efficiency. The discrete Fourier transforms of the time series and wavelet function can be multiplied together point-wise, and then inverse Fourier transformed. From this convolution, both the real $\Re(U_t(s))$ and imaginary $\Im(U_t(s))$ parts can be obtained which can be used to compute phase (ϕ).

$$\phi_t(s) = tan^{-1}\left(\frac{\Im(U_t(s))}{\Re(U_t(s))}\right) \tag{2}$$

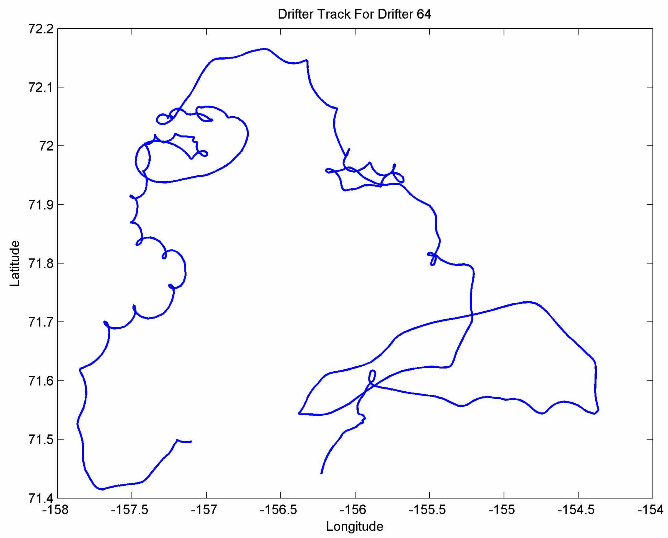

Figure 6: Movement of Drifter 64 (2012). Notice strong inertial movement around 71.8^o north and 157^o west.

Methods

Initially, these data were explored by plotting drifter tracks through space (Figure 6, e.g.), dividing the series into chunks of between two to five days, and computing rotary spectra (Figure 7), which use both the Fourier transforms of both east-west and north-south velocity to estimate the strength and period of rotational motion. This allowed the rotational motion of the drifters to be tracked through time. Details about the calculation and computation of rotary spectra have been omitted because ultimately they were not used in this analysis; however, they are an important part of the process and are something that will be a part of the continued analysis of these data because they respond to rotational motion and not just any periodic behavior.

Fourier analysis allows for the examination of time series in the frequency domain by using the Fourier transform to view a time series in terms of the strength of its periodic components. However, it comes with the limitation that it can only effectively be used for stationary time series. Inertial oscillation could be discerned using Fourier methods, but there would be no way of identifying how the signal changed over time. The short-time Fourier transform involves dividing the time series into many shorter series and doing a Fourier analysis on those smaller series. This allows for the examination of changing spectral

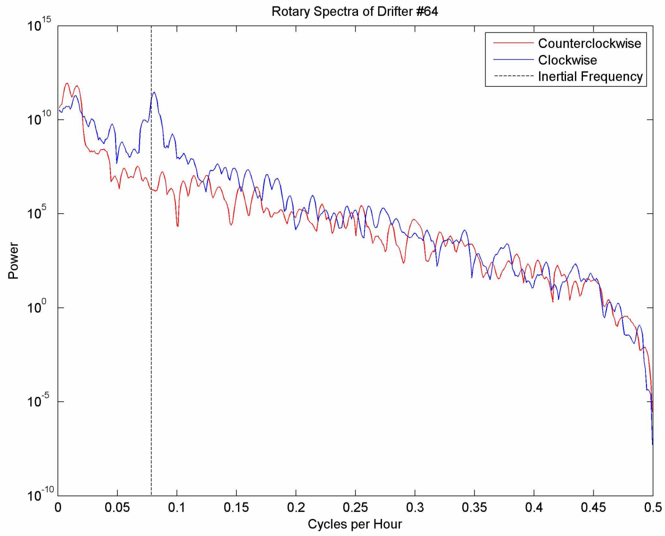

Figure 7: Calculated rotary spectra for drifter 64 (2012). Rotary spectra use both East-West and North-South velocity records to calculate frequency of rotational motion. This is particularly useful in the analysis of stationary time series, but the wavelet transform provides much of the same information without the requirement of stationarity.

power over time, but does not have the resolution of the wavelet transform (Mallat 1999, Elsayed 2010). Wavelets can be used to examine non-stationary time series to show how the frequency domain of a particular series may change over time (Daubechies 1990).

There are several considerations to make before choosing a specific type of wavelet for use in analysis. First, the choice between complex or real-valued wavelets is important depending on whether or not phase is of interest. Additionally, real-valued wavelets are better at recognizing discontinuity in the time domain. Next, width and shape are important. The shape of the wavelet is ideally chosen to resemble the types of periodic events the researcher is hoping to capture (Torrence & Compo 1998). The width of the wavelet is important because as the width increases the resolution in the time domain decreases. If a wavelet is wider and so takes into account more of the time series, it is more difficult to discern precise timing for changes in power. However, as the width of a wavelet increases, the resolution of the transform in the frequency domain increases. Ideally, a balance will be struck between resolution in time and frequency. There is the additional consideration of orthogonal and non-orthogonal wavelets, but most of the commonly used wavelets in physical science, including the Morlet and DOG are non-orthogonal. Though they can contain duplicated information, especially at low frequencies, they work well when sharp changes in wavelet power are not expected (Torrence & Compo 1998).

There are many different types of wavelet bases. For this analysis I used the Morlet wavelet, which is common for oceanography and other physical sciences. The Morlet wavelet (Figure 8) is defined as:

$$\Psi(x) = (\pi)^{-1/4} e^{ifx} e^{\frac{-x^2}{2}} \tag{3}$$

where f is the wavelet center frequency, which is set equal to 6 to satisfy the admissibility condition of wavelets (Farge 1992). The admissibility condition is a property of wavelets that causes the Fourier transform of the wavelet function to equal 0 at the frequency 0, and the integral of the wavelet function to be zero.

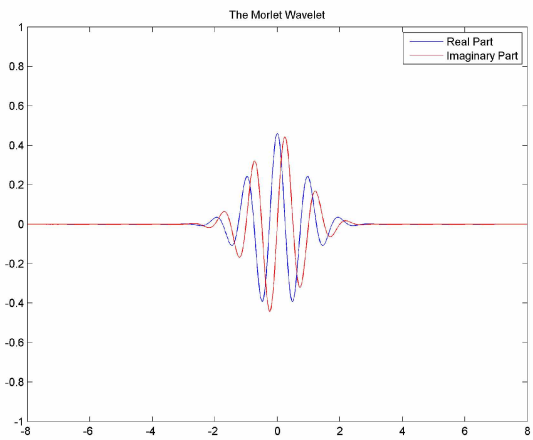

Figure 8: The real and imaginary parts of a basic Morlet wavelet.

Notice that the Morlet wavelet is defined as a complex-valued function. As was previously mentioned, not all wavelets are complex. The DOG (Derivative of Gaussian) wavelet, for example, is real-valued. When viewed in the frequency domain, real-valued wavelets have peaks in both the positive and negative direction, which allows them to recognize positive and negative oscillations in a given time series, while losing any information on phase (Emery & Thomson 2014, Torrence & Compo 1998). For applications in physical oceanography, complex wavelets are generally used since phase is an important characteristic of tides.

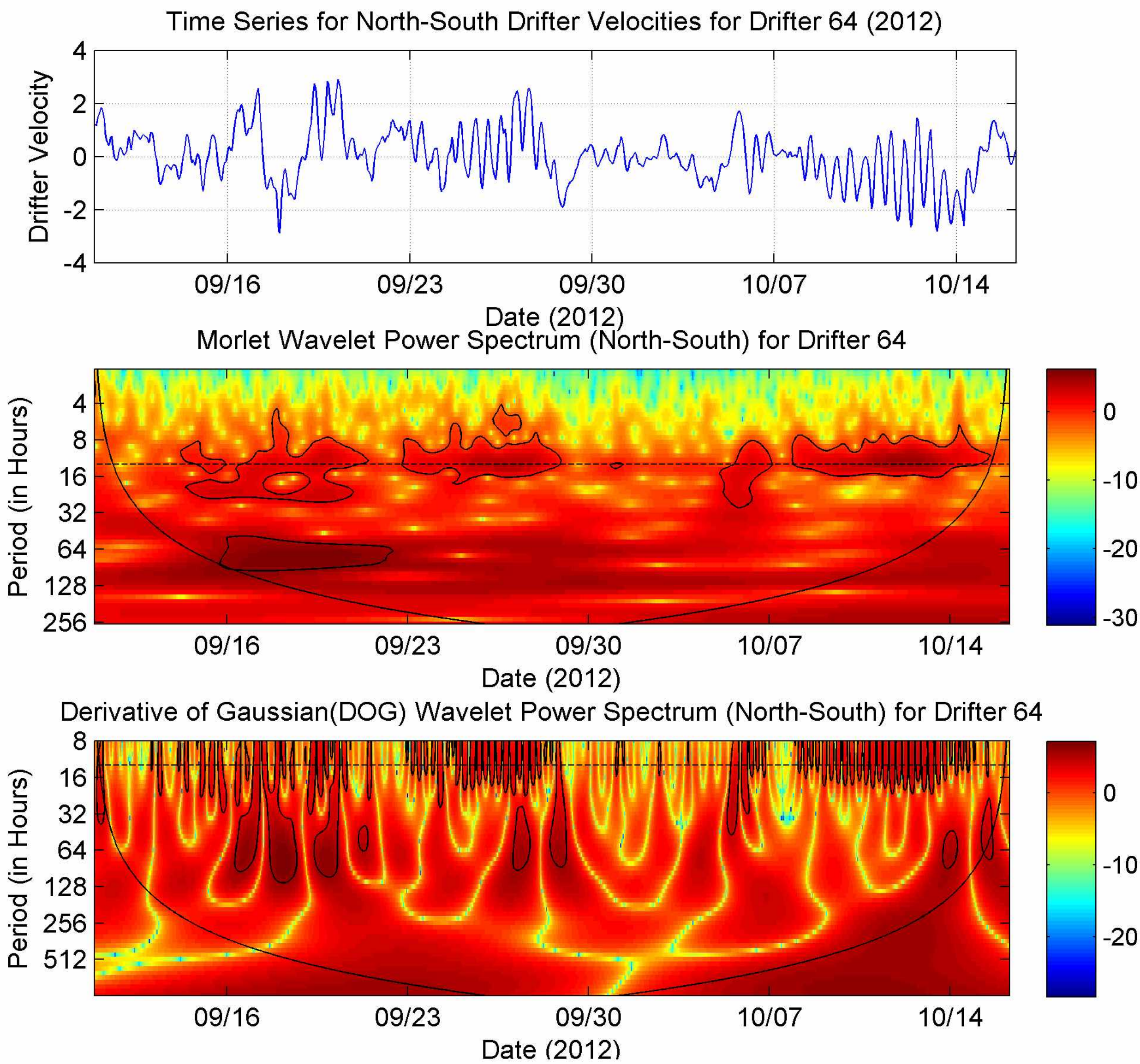

Figure 9: The North-South(U) Time Series for Drifter 64 in 2012 and the Plotted Morlet Wavelet transform and Derivative of Gaussian(DOG) Wavelet transform. The dotted line represents the period of inertial oscillation. Darker colors with black outlines show statistically significant power, which is where the power spectrum deviates enough from the background noise that we are 95% confident it is not by chance. If there is significant power at the inertial frequency, inertial oscillation is assumed.

The DOG and Morlet wavelets give similar results, though the DOG wavelet tends to give more spread on the period axis. This comparison (Figure 9) shows the similarity of these two wavelet transforms on the same time series. They both show inertial oscillation events in about the same spots on the time axis, though the Morlet wavelet gives more precise estimates of period.

Scale and period are very similar, but not exactly the same. When setting up a wavelet transform, scales need to be chosen. This is essentially setting the period of the wavelet that is going to move through the time series. As the convolution happens, whenever a period in the time series matches well with the scale of the wavelet, the wavelet transform will show significant power. Then the wavelet is set to the next scale and goes through the time series again checking for periodic behavior that matches with the new scale. However, the scale of the wavelet and the period in the time series are not quite equal, since the period refers to a Fourier period. So there is a conversion factor to convert from scale to Fourier period. This conversion factor is conditional on the type of wavelet. For the Morlet wavelet, it is approximately 1.033.

For each drifter, the velocity records were standardized by subtracting their means and dividing by their standard deviation, then the Morlet Wavelet transform for both the east-west(U) and north-south(V) velocities was computed in the frequency domain. One of the convolutions was calculated using a scale equal to a period of 12.745 hours. This is the period of inertial oscillation at latitude 70.31^o North, which is close to the mid-latitude point of the Chukchi Sea.

Two new time series were thus calculate, one containing the wavelet power, or strength of the signal, at approximately the period of inertial oscillation for the east-west velocity record and for the north-south velocity record. It is then necessary to choose an appropriate background spectrum to test whether or not the wavelet power spectrum significantly deviates from the chosen background spectrum. Verified with Monte Carlo simulations, Torrence & Compo (1998) show that given our standardized Fourier transformed time series, both the real and imaginary parts of $| U_t(s) |^2$, taken separately and squared, are each chi-square distributed with one degree of freedom, and so $| U_t(s) |^2$ is distributed as chi-square with two degrees of freedom. Using this chi-square distribution as a measure of statistical significance allows for a quantitative way deal with inertial events, both in number and in duration.

As previously mentioned, in determining significance in the spectral domain, it must be known what sort of background spectrum that data is from. The chi-square test tests if spikes in power at a particular frequency deviate significantly from this background spec-

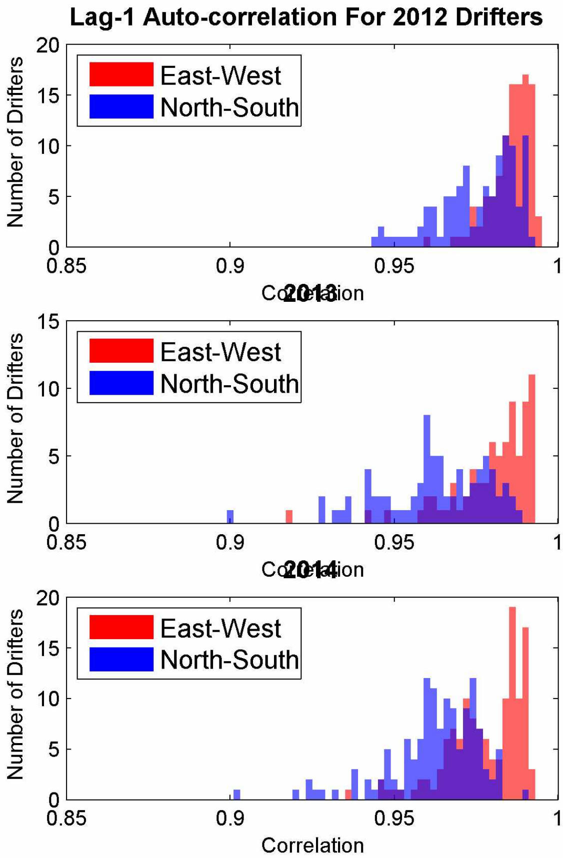

Figure 10: A histogram of lag-1 auto-correlation for all drifters with series length of greater than 100 hours. This was used in the determining of background AR(1) red noise spectrum auto-correlation. To reduce false positives, I used .95 as it was the lower end of common lag-1 auto-correlations.

trum. Fortunately, oceanic tides are a well documented red-noise process (Emery & Thomson 2014). White noise, which amounts to random errors around a common mean, is the most familiar background noise. Red noise however may be more familiar to some people as an auto-regressive order 1 (AR(1)) process, where measurements 1 unit apart have a certain auto-correlation. When this lag 1 auto-correlation is large, it can mean that values far from each other can still have significant correlation.

For this analysis, the auto-correlation functions (Figure 10) of all of the time series over all three years were examined. Lag-1 auto-correlation was often between .95 and 1, but again in the interest of minimizing false positives, lag-1 auto-correlation of .95 was used for the $AR(1)$ red noise background. Setting this value at the low end of common auto-correlations (Figure 10) reduces false positives, because as the background noise auto-correlation is raised, smaller values of signal strength become significant (Figure 11). Since much of this analysis is investigating relative characteristics and since changing the auto-correlation affects all years similarly, this choice seems appropriate.

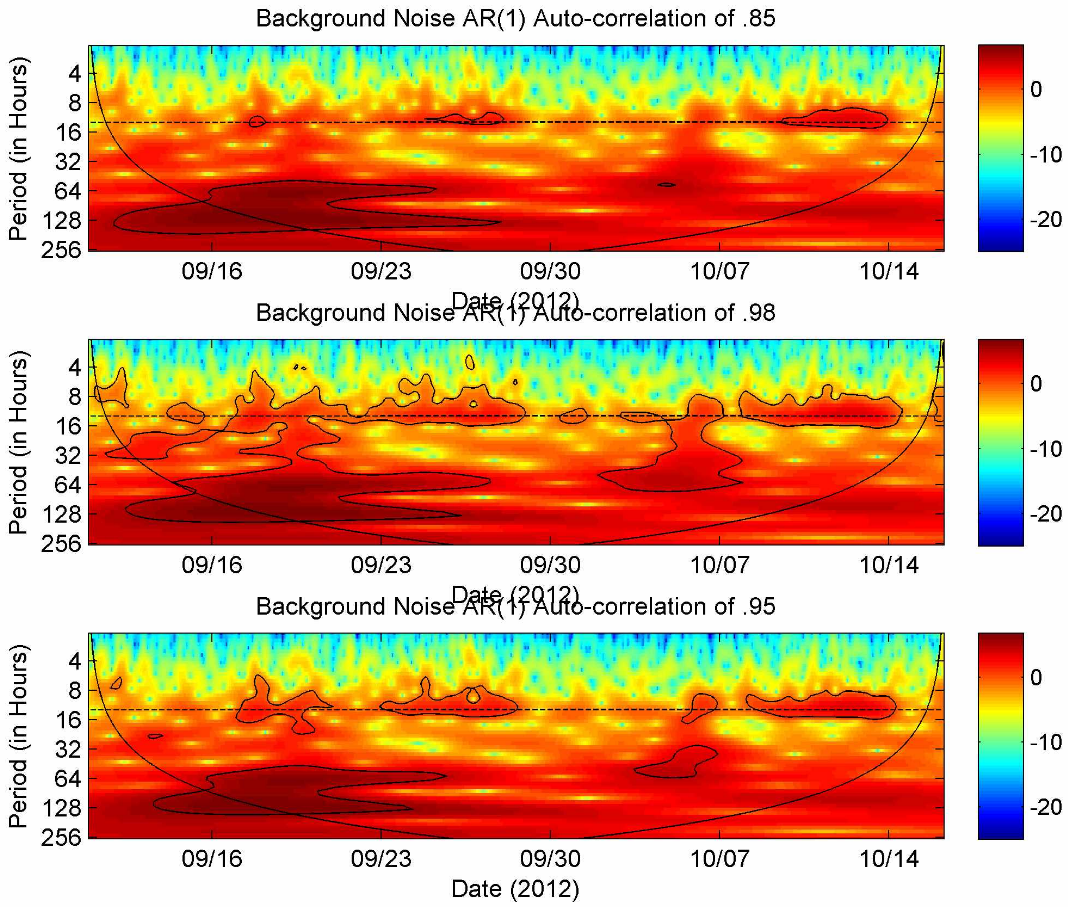

Figure 11: A comparison of the significance obtained by using different background red noise spectra. A value too low (such as .85 (top)) and only the strongest inertial oscillation events are significant. A value too high (.98 (middle)) and the wavelet transform picks up more noise and some M_2 tides as significant. A lag-1 auto-correlation (bottom) seems just about right.

When calculating the wavelet transform, a Fourier transform is computed as a faster method of convolution. The Fourier transform assumes that the time series is cyclical, and so would have miscalculations at the edge. But this seasonal drifter data is not cyclical, as at the end of each year the Chukchi Sea is covered in ice. To compensate for the non-cyclical nature of the data, the time series is padded with zeroes. However, during the convolution, some of the zeroes are multiplied with the wavelet and give poor estimates for certain periods. The cone of influence is a bowl shaped line visible in the plotted wavelet transforms, which shows where the calculated transform is trustworthy and where it is not (Figure 5 and bottom two panels of Figure 9). The cone of influence grows wider as period increases, because in order to capture long periods, the wavelet becomes very wide and so is more often multiplied by the padded zeroes. "Outside" the bowl means that calculated transform may be untrustworthy, as it has been contaminated with zeroes.

Results

The major results of this analysis are:

- In the Chukchi Sea, north-south velocity records detect inertial oscillation better than east-west velocity records
- The distributions of inertial oscillation event lengths in 2012 and 2013 were similar, while the distributions in 2014 had slightly thicker tails, increasing the mean inertial oscillation event length.
- For each subsequent year, both number of inertial oscillation events and proportion of time drifters spent in an inertial oscillation event increased.

Throughout all three years, the difference in number of inertial oscillation events between the east-west and north-south velocity records is fairly constant (Figure 12). Additionally, the ratio of time a drifter spent in an inertial oscillation event between the east-west and north-south velocities is also constant (Figure 13). Within each year, there were more inertial oscillation events captured by the north-south velocity record than by the east-west velocity record. However, the distributions of inertial oscillation event lengths between north-south and east-west velocities are very similar (Figure 16).

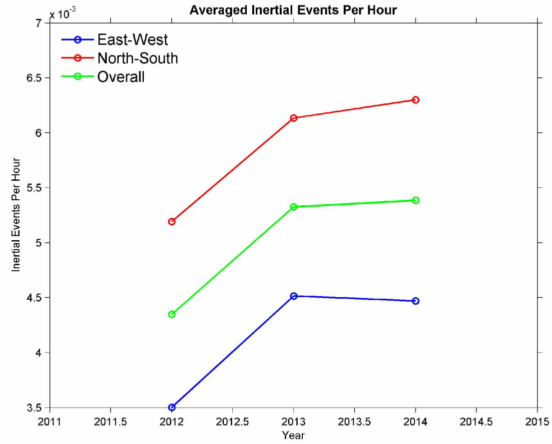

Figure 12: A year by year look at the total number of inertial events. There is a definite increase in number of events from 2012 to 2013, but the numbers remained fairly constant from 2013 to 2014. Notice that in each year, the north-south velocity records captured many more inertial events than the east-west velocity records.

There seems to be (Figures 12&13) an upward trend toward increased inertial oscillation events and drifters spent longer amounts of time in inertial oscillation events in both velocity records in all three years. Because there are only three years to examine at this point, it is difficult to tell whether this is an actual trend, part of cyclic behavior, or just random noise. The collection of several more years of data could shed more light on the way inertial oscillation is changing in the Chukchi Sea.

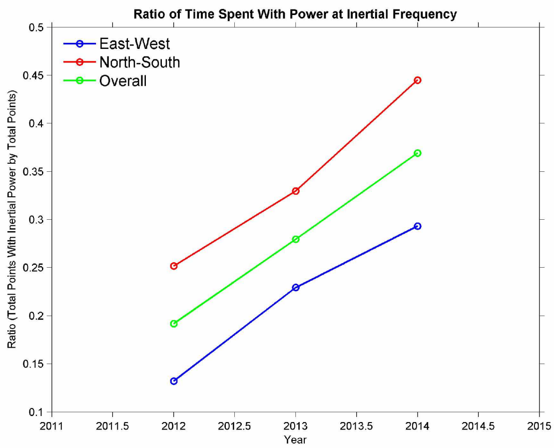

Figure 13: The total amount of time that all drifters in a given year spend with significant power at the inertial frequency. There is an obvious trend upwards in this case, though because we only have three years worth of data, it is difficult to say whether it is an actual trend or just random noise. This does imply that inertial oscillation events were longer on average in 2014 than in 2013, since the number of events in those years were very similar.

From 2013 to 2014, the total number of inertial oscillation events did not increase much, but the total time spent in an inertial oscillation event did increase. This suggests that inertial oscillation events lasted longer in 2014 than in 2013.

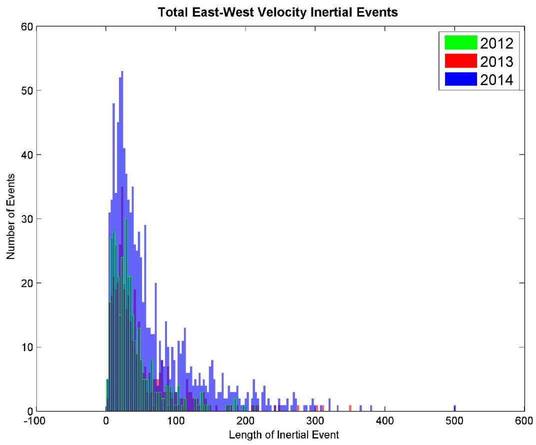

Figure 14: A histogram of total number of east-west events for a given event length in hours. The histograms have very similar shapes, with 2014 having significantly more events at almost every length. The histogram for 2014 does not taper quite as quickly either, implying longer events on average.

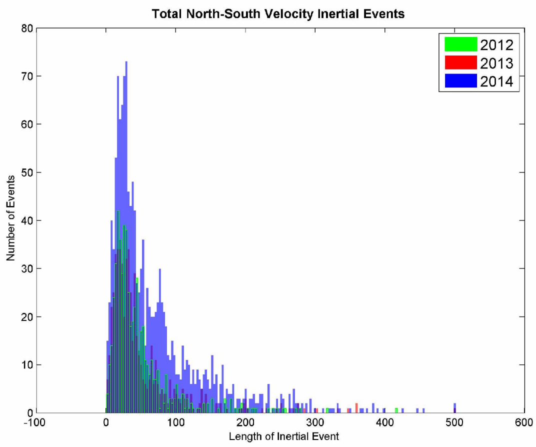

Figure 15: A histogram of total number of north-south events for a given event length in hours. This shows very similar results to the east-west velocity record (Figure 14), except there are more events at each length and there are quite a few more events of greater than 200 hours.

For both east-west and north-south velocities, 2014 had more inertial oscillation events of nearly all lengths than either 2012 or 2013 (Figures 14 & 15). This is partially because more drifters were deployed in 2014; however, 2014 had more events even when normalized by total number of data points (Figures 12 & 13).

Table 1: The mean north-south and east-west inertial oscillation event length for the years 2012, 2013, and 2014.

Year	N-S Mean	E-W Mean
2012	47.9	37.28
2013	52.04	49.21
2014	70.47	65.22

In both figures 14 and 15, the 2012 and 2013 histograms exhibit almost perfect overlap. Since 2013 had fewer drifters and fewer data points on the whole, that leads to a higher normalized event total, which was seen previously.

These histograms also confirm that the average length of an inertial oscillation event is much longer is 2014 than 2013 or 2012. From year to year, the shape of the histograms is very similar, however the 2014 histogram drops off more gradually and has a slightly thicker tail. These data are not independent, so there is no easy way to put the mean in a confidence interval, but the large difference in average event length from the first two years to 2014 is noteworthy.

Discussion

The upward trend in both the number of continuous inertial oscillation events and in total percentage of time spent in inertial oscillation may be significant. However, three years worth of data amounts to three data points when examining this trend, and is like looking through a keyhole. The trend is visually striking, but it seems unlikely to sustain. As more and more of the drifter time series is spent in inertial oscillation events, these events will blend together, causing a larger ratio of time spent with inertial power, but few inertial oscillation events. If, for example, a drifter spent all of its time rotating in an inertial oscillation event, it could only have experienced one continuous inertial oscillation event.

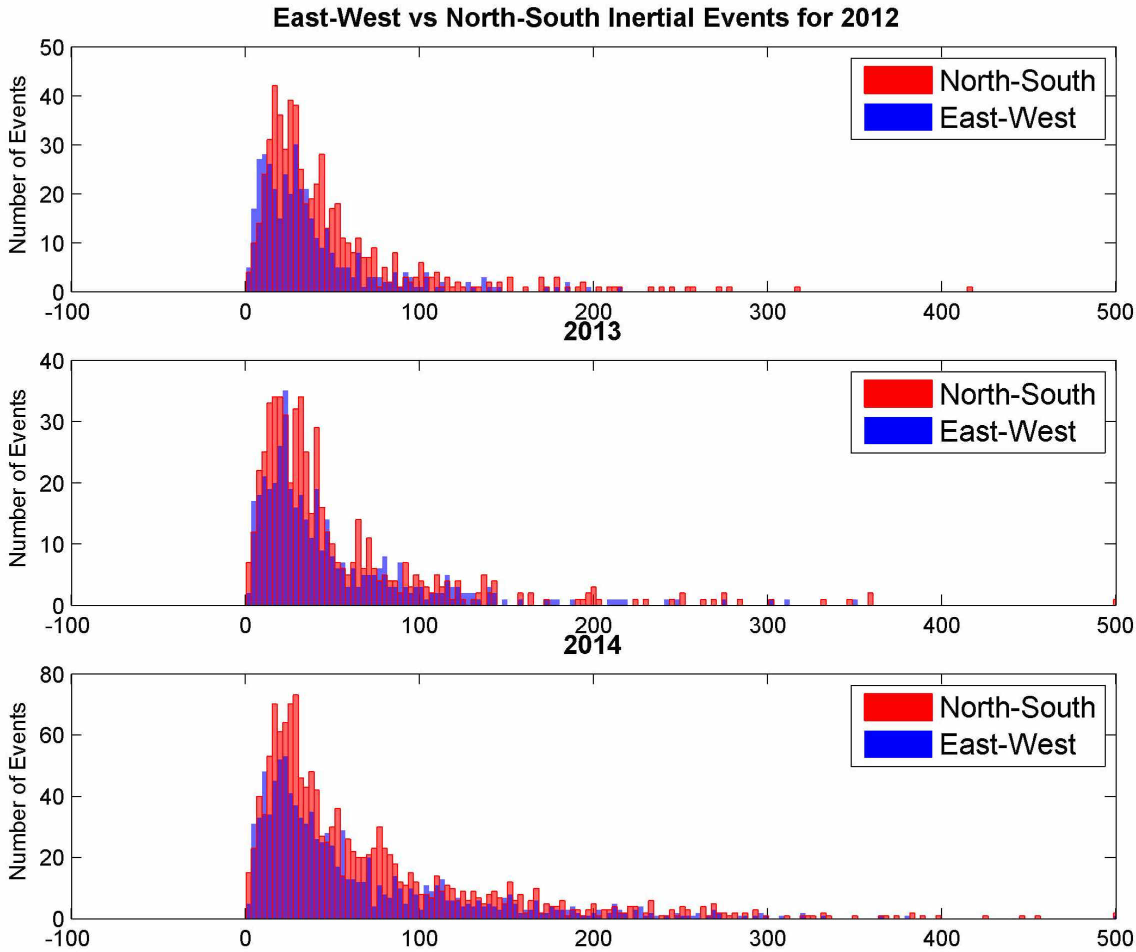

Figure 16: A year by year histogram comparison for length in hours and number of north-south and east-west capture inertial events. In each year there are many more inertial oscillation events captured by north-south velocity records than the east-west velocities. The distribution of events lengths is pretty consistent from year to year and from east-west velocities to north-south velocities, except that the north-south histograms seem to have slightly fatter tails, which is most notable in 2014.

The cause of this trend is unclear as is its trajectory. The Chukchi Sea could be in the midst of a transition period. With climate changing so rapidly in the arctic, disruptions to once typical environmental conditions are to be expected. Another possibility is that inertial oscillation events in the Chukchi are a cyclical process. After 2014, perhaps there will be a downturn in both number of inertial oscillation events and percentage of time spent in an event. It seems possible the retreating sea ice is causing changing currents which increases the number of inertial oscillation events. It could also be that climate change is bringing more storms through the Chukchi Sea, which are strumming the waters in such a way that is increasing the occurrences of inertial oscillation. Those possibilities could be examined by

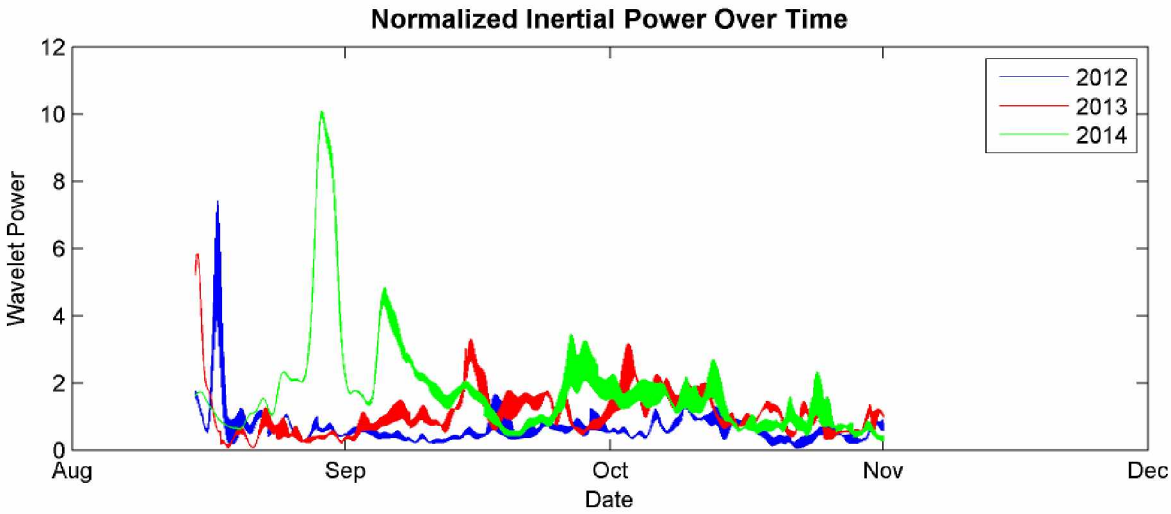

Figure 17: Total normalized wavelet power at the inertial frequency for all drifters that transmitted for longer than about 4 days. This was used to examine similarities in the timing of strong inertial oscillation events. For example, was there a certain time of year when very strong inertial oscillation events were more prevalent. Without more work, this is inconclusive.

further investigation into how currents are changing, how currents affect inertial oscillation, and how storms and winds are shifting. This could also be just random noise around a typical fall in the Chukchi Sea. Each additional year of data will provide more insight of the overall trend.

It is unlikely the differences in inertial oscillation between north-south and east-west velocities is random noise. For three years in a row, the number of inertial events and the ratio of time a drifter spent in inertial motion was significantly higher for the north-south velocities than the east-west velocities. Furthermore, the difference between them remained fairly constant. It is unlikely this difference is due to randomness and more likely due to physical properties of Chukchi that may cause inertial oscillation to be more predominant in certain directions. There are several possible explanations for this. The Chukchi Sea has prevailing northeasterly winds (Hopcroft et al., 2008). It is possible this makes a particular interaction with currents only happen in a certain direction. It is also possible the drifter spends more time traveling in a particular direction which lends itself to being better captured by north-south velocities.

Over the three years represented in this data set, there is also a trend toward longer inertial oscillation events. This is most easily seen in the histograms (Figure 16), but is also plain from (Table 1), which shows an increase from 2012 to 2014 of over 150%. It is easy to speculate that these inertial motions are stronger or weaker from year to year, but there is no striking visual evidence of year to year correlation in wavelet power surges (Figure 17). An analysis of Markov chains along with a more in depth examination of cross-correlation could shed light on this question.

Conclusion

Wavelets provide a useful tool in the analysis of inertial oscillation and other non-stationary processes. In terms of statistical analysis, wavelets are somewhat undeveloped for quantitative analysis, such as hypothesis tests. They can be used to focus an investigation on times, periods, or phases where interesting behavior of the time series may be occurring.

This analysis was largely exploratory and yielded few concrete answers, but did shed light on processes at work in the Chukchi Sea and ways to approach them in future work. As a way to improve this analysis, examining better ways to handle the missing data would be useful. In this case they were simply set to 0, which is a sub-optimal way to deal with them. Also, finding a way to look at year to year differences that come from the use of different quality data over the years could also help improve this study.

The literature on inertial oscillation shows that a sudden shift in winds acting on water traveling in a particular direction is largely what causes inertial oscillation (reviewed in Stewart 2008). This is a necessarily simplistic view as there are many complex factors at play. But a detailed spatio-temporal model that takes into account wind data would greatly help the understanding of inertial oscillation in the Chukchi Sea. Additionally, a visualization of periodic behavior using short-time rotary spectra could give additional information on inertial oscillation since non-rotational periodic components like the M_2 tidal constituent would be filtered.

Inertial oscillation is not generally included in discussions about tidal constituents because it is sporadic and varies in strength and period from place to place. But it is an influential component of our oceans and seas. As the Chukchi Sea continues to change, it is important that we learn how it works. How will loss of sea ice affect physical, biological, and chemical processes? How will warming of the water affect these things? How will oil propagate if there is a spill? How is nutrient dispersal changing? Understanding inertial oscillation in the Chukchi Sea is complementary and arguably essential to the understanding of all these questions.

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